should be taken only as provisional, enabling us to go on and examine the general properties of the curves. The ordinates beyond the limits x_1 are not zero, though they may be small enough to be neglected without material error. I think that the decimal coefficients in (41) are very nearly if not entirely correct, but am not so sure of those in (37), because so far as we can judge from Table III, the values of Y beyond the adopted lower limit $-x_1 = -37$ nearly, will not be very small. It is to be hoped that the integrals of (33) and (40) may yet be evaluated rigorously between the limits $\pm\infty$, so as to give the constants of integration for these curves in an exact form, as has been done in the simpler and well known case of the probability curve (31).

The last one of the three conditions in (40) is satisfied necessarily, because the curve is symmetrical with respect to the axis of Y, and thus its equation (41) shows only two constants of integration. In all cases where the order n of the general differential equation (28) is odd, the curve is symmetrical, the alternate conditions

$$\int_{-\infty}^{\infty} x^3 y dx = 0, \qquad \int_{-\infty}^{\infty} x^5 y dx = 0, \qquad \int_{-\infty}^{\infty} x^7 y dx = 0, \text{ \&c.,}$$

are necessarily satisfied, and the number of constants of integration to be determined is only $\frac{1}{2}(n+1)$.

ON THE VARIATION IN THE LENGTH OF THE DAY.

BY PROFESSOR DANIEL KIRKWOOD.

IF we let m = the mass of a rotating globe in the process of condensation;
t = its time of rotation;
k = its principal radius of gyration;
r = its radius, and
ω = its angular velocity; then the principle of the preservation of areas gives us

$$m\omega k^2 = \tfrac{2}{5}m\omega r^2 = c = \text{a constant.}$$

$$\therefore\ \omega = \frac{5c}{2m}\cdot\frac{1}{r^2}; \text{ or, since } m \text{ is constant,}$$

$$\omega = \frac{c'}{r^2};$$ that is, the angular velocity varies inversely as the square of the radius.

If now we let t', r' and ω' represent the time, radius and angular velocity for any other epoch, we shall have

$$\omega : \omega' :: r'^2 : r^2; \text{ and}$$
$$\omega : \omega' :: t' : t;$$
$$\therefore t : t' :: r^2 : r'^2;$$

or, the time of rotation varies as the square of the radius. This proportion gives us

$$t' = t\left(\frac{r'}{r}\right)^2.$$

We may apply this formula to determine what was the length of the day when the earth's radius was 100 miles greater than at present.

$$t' = 23^{h}.933\left(\frac{4056}{3956}\right)^2 = 25^{h}9^{m}29^{s}.$$

When the radius was about 21,500 ms. the day was equal to the moon's period of revolution, so that, during an indefinite period, the moon remained constantly on the same meridian. In this estimate the moon's mean distance from the earth is supposed to be constant and the effect of tidal retardation is disregarded. We find in the same manner that when the radius was about 75,000 miles the day and the year were equal. Finally, if the earth ever extended to the moon's orbit the period of rotation, by the same formula, was nearly ten years. So likewise when Mars filled the orbit of Phobos, his rotation period was $7^{d}.65$, or 24 times the orbital period of the satellite. It is probable, however, that in the gaseous form all parts of the mass would not rotate in the same time.

In the case of the earth or in that of any planet attended by satellites, the acceleration due to contraction would probably compensate, or more than compensate, any tidal retardation.* As each of these opposing forces, however, has evidently been variable, the determination of their relative effects is a problem of great difficulty.

[If it be true that the several planets of the solar system were detached from a central rotating gaseous mass, as is assumed in the Nebular Hypothesis, then it appears, by application of the above formula, that when the rotating mass extended to the orbits of the several planets the angular velocity of its equatorial surface must have been much less than that of its central portion, so that it is possible the rings, which subsequently formed several, or all, of the planets, may have been detached simultaneously.—Ed.]

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*The effects of the tidal retardation have been ably discussed by Delaunay, Adams, G. H. Darwin, and others, but the data are insufficient to justify any definite conclusion.